

# MATHEMATICAL FOUNDATIONS OF PROBABILISTIC THEORY OF INVESTMENT

## Syllabus

Measures of investment risk: variance, semi-variance, shortfall probability, expected shortfall, value at risk.

Modern portfolio theory. Expectation and variance of the portfolio return. Minimum variance portfolio. Choice of optimal portfolio. Benefits of diversification.

Models of assets return. Multifactor models. Macroeconomic, fundamental and statistical factors. One-factor model.

Capital asset pricing model. Separation theorem. Market price of risk. Restrictions of CAPM. Arbitrage pricing theory.

Effective market hypothesis. Strong, semi-strong and weak forms of EMH. Informational efficiency. Volatility tests.

Stochastic models of asset price. Lognormal model. Wilkie model. Calibration of models. Brownian motion and stochastic calculus. Basic facts and notions of the theory – Wiener process, martingals, Ito integral, Ito formula, stochastic differential equations.

Introduction to derivative pricing. Absence of arbitrage opportunity. Self-financing strategies.

Complete market. Price of the forward contract. Estimates of option prices. Put-call parity.

Binomial model. Risk-neutral measure. Recombined binomial tree. State-price deflator approach.

Continuous time models. Cameron-Martin-Girsanov theorem.

Black-Scholes model. Black-Scholes formula. Black-Scholes partial differential equation.

Garman- Kohlhagen formula.

Further analysis of Black-Scholes model. Greeks.

Time structure of interest rates. Vasicek, Cox-Ingersoll-Ross and Hull-White models.

## REFERENCES

### a) primary:

- 1) Cairns A. J. Interest rate models: an introduction. Princeton University Press, 2004.
- 2) Elton E. J., Grube, M. J., Brown, S. J., Goetzmann, W. N. Modern portfolio theory and investment analysis. – John Wiley & Sons., 2009.
- 3) Luenberger D. G. Investment science – Oxford University Press, 1998.

### b) additional:

- 1) Karatzas I., Shreve S. E. Methods of Mathematical Finance. Springer, 1998.
- 2) Shiryaev A.N. Essentials of stochastic finance. World Scientific. Singapore, 1999.